Separation properties on p*gp-closed sets in topological spaces

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Abstract: The purpose of this article is to investigate the spaces namely p*gp-T_0 spaces, p*gp-T_1 spaces, and p*gp-T_2 spaces utilizing p*gp-open sets. Also we discuss their relationship with already existing concepts and their properties. Moreover, we establish p*gp-D_k (k = 0, 1, 2) spaces using p*gp-open sets and investigate their properties. Finally, we discuss their relationship with p*gp-T_k (k = 0, 1, 2, ½) spaces and p*gp-D_k (k =0, 1, 2) spaces.

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1. Introduction

The study of generalization of closed sets has been found to ensure some new separation axioms which are very useful in the study of certain objects of digital topology. Separation axioms play a dominant role in the classification of topological spaces and they are of varied of strengths. This notion has been studied extensively in recent years by many topologists. These axioms are called T_0, T_1 and T_2 in ascending order of strength, T_0 is the weakest separation axiom. T_0-axiom was introduced by Kolmogorov **[12]** in 1933 and T_1-axiom was intiated by Frechet in 1907. Hausdorff brought out T_2-axiom in 1923. The class of $T_{1/2}$ spaces lies between the class of T_0 spaces and the class of T_1 spaces. The notions of D-type separation axioms are also available in the literature of topology. In 1982, Tong **[22]** introduced the notion of D sets by using open sets and used the notion to define D_i spaces. In 2001, Jafari **[6]** brought out weak separation axioms. In 1970, Levine **[13]** discovered the concept of generalized closed sets (briefly g-closed). In 1996, Maki, Umehara and Noiri **[14]** have been introduced the class of pre generalized closed sets and used them to obtain properties of pre-$T_{1/2}$ spaces. Selvi **[20]** proposed pre*closed sets using the g-closure operator due to Dunham **[3, 4]**. Andrijevic **[1]**, Gnanambal **[5]**, Njastad **[17]**, Mashhour **[15]**, stone **[21]**, Dontchev **[2]**, Veerakumar **[23, 24]**, Nagaveni **[16]**, Zaitsev **[25]**, Park **[11, 18]**, Sarsak and Rajesh **[19]**, put forward the concepts of semi pre-closed, gpr-closed, α-closed, pre-closed, regular-closed, gsp-closed, g*p-closed, pre semi closed, wg-closed, rwg-closed, gp-closed, π-closed, πgp-closed and πgsp-closed respectively.

The authors **[7, 8, 9, 10]** brings out the p*gp-closed sets, p*gp-open sets, p*gp-continuous functions, p*gp-irresolute functions, p*gp-closed maps and p*gp-open maps in topological spaces and established their relationships with some generalized sets in topological spaces. In this article, p*gp-T_k (k = 0, 1, 2, 1/2) spaces and p*gp-D_K (k = 0, 1, 2) spaces in topological spaces are introduced and their properties are investigated. The characterizations for these spaces and relationship among themselves and with other known separation axioms are examined.

2. Preliminaries

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, τ) will be replaced by X if there is no changes of confusion. For a subset A of a topological space X, cl(A), int(A) and X\A denote the closure of A, the interior of A and the complement of A respectively. We recall the following definitions and results.

Definition 2.1. [13] Let (X, τ) be a topological space. Then the subset A of X is said to be

(i) generalized closed (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is an open in (X, τ).
(ii) generalized open (briefly g-open) if its complement, X\A is g-closed.

Definition 2.2. Let (X, τ) be a topological space and A ⊆ X. The generalized closure of A **[4]**, denoted by cl*(A) and is defined by the intersection of all g-closed sets containing A and generalized interior of A **[4]**, denoted by int*(A) and is defined by union of all g-open sets contained in A.

Definition 2.3. Let(X, τ) be a topological space and A ⊆ X. Then
(i) A is α-open if A ⊆ int(cl(int(A))) and α-closed if cl(int(cl(A))) ⊆ A **[17]**.
(ii) A is pre open if A ⊆ int(cl(A)) and pre closed if cl(int(A)) ⊆ A **[15]**.
(iii) A is pre*open if A ⊆ int*(cl(A)) and pre*closed if cl*(int(A)) ⊆ A **[20]**.
(iv) A is regular open if A = int(cl(A)) and regular closed if A = cl(int(A)) **[21]**.
(v) A is semi pre open if A ⊆ cl(int(cl(A))) and semi pre closed if int(cl(int(A))) ⊆ A **[1]**.
(vi) π-closed set **[25]** if A is a finite intersection of regular closed sets. The complement of a π-closed set is called a π-open set.

Definition 2.4. Let (X, τ) be a topological space and A ⊆ X. Then the
(i) pre closure of A **[15]** is defined as the intersection of all pre closed sets containing A.
(ii) semi pre closure of A **[1]** is defined as the intersection of all semi pre closed sets containing A.

Definition 2.5. Let (X, τ) be a topological space. A subset A of X is said to be
1. generalized pre closed set (briefly gp-closed) **[11]** if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2. Generalized*pre closed set (briefly g*p-closed) **[23]** if pcl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
3. weakly generalized closed set (briefly wg-closed) **[16]** if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).
4. generalized pre regular closed set (briefly gpr-closed) **[5]** if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).
5. generalized semi pre closed set (briefly gsp-closed) **[2]** if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
6. pre semi closed set **[24]** if spcl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
7. πgp-closed set **[18]** if pcl(A) ⊆ U whenever A ⊆ U and U is π-open in (X, τ).
8. regular weakly generalized closed set (briefly rwg-closed) **[16]** if cl(int(A)) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).
9. π-generalized semi pre closed set **[19]** (briefly πgsp-closed) if spcl(A) ⊆ U whenever A ⊆ U and U is π-open in (X, τ).

The complements of the above mentioned closed sets are their respective open sets.

Theorem 2.6. [20] Let (X, τ) be a topological space. Then
(i). Every closed set is pre*closed.
(ii). Every open set is pre*open.

Definition 2.7. [7] A subset A of a topological space (X, τ) is said to be pre*generalized pre closed (briefly p*gp-closed) if pcl(A) ⊆ U whenever A ⊆ U and U is pre*open in (X, τ). The collection of p*gp-closed sets of X is denoted by p*gp-C(X).

Theorem 2.8. [7] Let (X, τ) be a topological space. Then
(i) Every closed set is p*gp-closed.
(ii) Every regular closed set is p*gp-closed.

(iii) Every α-closed set is p*gp-closed.
(iv) Every pre closed set is p*gp-closed.
(v) Every p*gp-closed set is gp-closed, gpr-closed, wg-closed, rwg-closed, πgp-closed, gsp-closed, πgsp-closed, pre semi closed, g*p-closed.
(vi) Intersection of any two p*gp-closed sets is p*gp-closed.

Definition 2.9. [8] A subset A of a topological space (X, τ) is said to be pre*generalized pre open (briefly p*gp-open) if its complement is p*gp-closed in X. The collection of p*gp-open sets of X is denoted by p*gp-O(X).

Theorem 2.10. [8] Let (X, τ) be a topological space. Then
(i). Every open set is p*gp-open.
(ii). Every regular open set is p*gp-open.
(iii). Every α-open set is p*gp-open.
(iv). Every pre open set is p*gp-open.
(v). Every p*gp-open set is gp-open, gpr-open, wg-open, rwg-open, πgp-open, gsp-open, πgsp-open, pre semi open, g*p-open.
(vi). Union of any two p*gp-open sets is p*gp-open.

Definition 2.11. [8] Let A be a subset of a topological space (X, τ). Then p*gp-closure of A is defined as the intersection of all p*gp-closed sets in X containing A. It is denoted by p*gpcl(A). That is p*gpcl(A) = $\cap$ {F: A $\subseteq$ F and F $\in$ p*gp-C(X)}.

Definition 2.12. [8] Let A be a subset of a topological space (X, τ). Then p*gp-interior of A is defined as the union of all p*gp-open sets contained in A. It is denoted by p*gpint(A).
That is p*gpint(A) = $\cup$ {V : V $\subseteq$ A and V $\in$ p*gp-O(X)}.

Theorem 2.13. [8] Let A be a subset of a topological space (X, τ). Then
(i) A is p*gp-closed if and only if p*gpcl(A) = A.
(ii) A is p*gp-open if and only if p*gpint(A) = A.

Theorem 2.14. [7] For every element x in a space X, X\{x} is p*gp-closed or pre*open.

Theorem 2.15. [8] For a topological space (X, τ), every singleton of X is either p*gp-open or pre*closed.

Definition 2.16. [9] A function f : (X, τ)$\rightarrow$ (Y, σ) is said to be
(i) p*gp-continuous if inverse image of each open set in Y is p*gp-open in X.
(ii) p*gp-irresolute if inverse image of each p*gp-open set in Y is p*gp-open sets in X.

Definition 2.17. [10] A function f : (X, τ)$\rightarrow$ (Y, σ) is said to be a

(i) p*gp-closed map if image of each closed set in X is p*gp-closed in Y.
(ii) p*gp-open map if image of each open set in X is p*gp-open in Y.

3. Pre*generalized Pre T_0 Spaces

In T_0 spaces, two distinct points x, y are separated by means of an open sets containing a specific point of x, y and not containing the other. In this section, we introduce p*gp-T_0 spaces and investigates their basic properties.

Definition 3.1. A topological space (X, τ) is said to be p*gp-T_0 if for each pair of distinct points x, y in X, there exists a p*gp-open set U such that either x $\in$ U and y $\notin$ U or x $\notin$ U and y $\in$ U.

Theorem 3.2. In any topological space,
(i) Every p*gp- T_0 space is gp- T_0.

(ii) Every p*gp- T_0 space is gpr-T_0.
(iii) Every p*gp-T_0 space is wg-T_0.
(iv) Every p*gp-T_0 space is rwg-T_0.
(v) Every p*gp-T_0 space is πgp-T_0.
(vi) Every p*gp- T_0 space is gsp-T_0.
(vii) Every p*gp-T_0 is space πgsp-T_0.
(viii)Every p*gp-T_0 space is pre semi-T_0.
(ix) Every p*gp-T_0 is space g*p-T_0.

Proof: (i) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is gp-open. Hence X is gp-T_0.

(ii) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp- T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is gpr-open. Hence X is gpr-T_0.

(iii) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is wg-open. Hence X is wg-T_0.

(iv)Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. By assumption, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is rwg-open. Hence X is rwg-T_0.

(v) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is πgp-open. Hence X is πgp-T_0.

(vi) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is gsp-open. Hence X is gsp-T_0.

(vii) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is πgsp-open. Hence X is πgsp-T_0.

(viii) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. By assumption, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is pre semi open. Hence X is pre semi-T_0.

(ix) Suppose X is p*gp-T_0 space. Let x and y be two distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set V containing one of x and y but not other. By Theorem 2.10 (v), V is g*p-open. Hence X is g*p-T_0.

Theorem 3.3. In any topological space,
(i) Every T_0 space is p*gp-T_0.
(ii) Every regular-T_0 space is p*gp-T_0.
(iii) Every α-T_0 space is p*gp-T_0.
(iv) Every pre-T_0 space is p*gp-T_0.

Proof: (i) Suppose X is T_0 space. Let x and y be two distinct points in X. Since X is T_0, there exists an open set V containing one of x and y but not other. By Theorem 2.10 (i), V is p*gp-open. Hence X is p*gp-T_0.

(ii) Suppose X is regular T_0 space. Let x and y be two distinct points in X. Since X is regular-T_0, there exists a regular open set V containing one of x and y but not other. By Theorem 2.10 (ii), V is p*gp-open. Hence X is p*gp-T_0.

(iii) Suppose X is α-T_0 space. Let x and y be two distinct points in X. Since X is α-T_0, there exists a α-open set V containing one of x and y but not other. By Theorem 2. 10 (iii), V is p*gp-open. Hence X is p*gp-T_0.

(iv) Suppose X is pre-T_0 space. Let x and y be two distinct points in X. Since X is pre-T_0, there exists a pre open set V containing one of x and y but not other. By Theorem 2.10 (iv), V is p*gp-open. Hence X is p*gp-T_0.

Remark 3.4. The converse of each statement in Theorem 3.2 and Theorem 3.3 are not true as shown in the following examples.

Example 3.5. Let X = {a, b, c} and τ ={ϕ, {c}, {b, c}, X}. Then, X is p*gp-T_0 but not T_0 and also gp-T_0, pre semi-T_0 and g*p-T_0 but not p*gp-T_0.

Example 3.6. Let X = {a, b, c} and τ = {ϕ, {c}, {a, b}, X}. Then, X is p*gp-T_0 but not regular-T_0.

Example 3.7. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then, X is p*gp-T_0 but not α-T_0 and also gpr-T_0 and πgsp-T_0 but not p*gp-T_0.

Example 3.8. Let X = {a, b, c} and τ = {ϕ, {c}, X}. Then, X is wg-T_0 and gsp-T_0 but not p*gp-T_0.

Example 3.9. Let X = {a, b, c} and τ ={ϕ, {a}, {b}, {a, b}, X}. Then, X is rwg-T_0 but not p*gp-$T_{0.}$.

Example 3.10. Let X = {a, b, c} and τ ={ϕ, {a}, {a, b}, X}. Then, X is πgp-T_0 but not p*gp-T_0.

Example 3.11. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then, X is p*gp-T_0 but not pre-T_0.

Theorem 3.12. A topological space (X, τ) is p*gp-T_0 if and only if for each pair of distinct points x, y in X, p*gpcl({x}) ≠ p*gpcl({y}).

Proof: Let X be a p*gp-T_0 space and x, y be any two distinct points of X. Then there exists a p*gp-open set U containing x or y, say x but not y. Since U is p*gp-open, X\U is a p*gp-closed set which does not contain x but contains y. Since p*gpcl({y}) is the smallest p*gp-closed set containing y, p*gpcl({y}) ⊆ X\U. Then x ∉ p*gpcl({y}). Hence p*gpcl({x}) ≠ p*gpcl({y}).

Conversely, suppose that x, y ∈ X with x ≠ y and p*gpcl({x}) ≠ p*gpcl({y}). Then there exists a point z ∈ X such that z ∈ p*gpcl({x}) but z ∉ p*gpcl({y}). Now, claim that x ∉ p*gpcl({y}). Suppose x ∈ p*gpcl({y}), then p*gpcl({x}) ⊆ p*gpcl({y}). This implies, z ∈ p*gpcl({y}), which contradicts z ∉ p*gpcl({y}). Therefore, x ∉ p*gpcl({y}). Since p*gpcl({y}) is p*gp-closed set containing y but not x, then X\p*gpcl({y}) is a p*gp-open set containing x but not y. Hence X is a p*gp-T_0 space.

4. Pre*generalized Pre T_1 Spaces

In T_1 spaces, two distinct points x, y are separated by means of an open sets containing each but not the other. In this section, we describe p*gp-T_1 spaces and investigates their basic properties.

Definition 4.1. A topological space (X, τ) is said to be p*gp-T_1 if for each pair of distinct points x, y in X, there exists two p*gp-open sets U and V such that x ∈ U but y ∉ U and x ∉ V and y ∈ V.

Theorem 4.2. In any topological space,

(i) Every p*gp-T_1 space is gp-T_1.
(ii) Every p*gp-T_1 space is gpr-T_1.
(iii) Every p*gp-T_1 space is wg-T_1.
(iv) Every p*gp-T_1 space is rwg-T_1.
(v) Every p*gp-T_1 space is πgp-T_1.
(vi) Every p*gp-T_1 space is gsp-T_1.
(vii) Every p*gp-T_1 is space πgsp-T_1.
(viii)Every p*gp-T_1 space is pre semi-T_1.
(ix) Every p*gp-T_1 is space g*p-T_1.

Proof: (i) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are gp-open sets. Hence X is gp-T_1.

(ii) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets U and V such that $x \in U$ but $y \notin U$ and $x \notin V$ and $y \in V$. By Theorem 2.10 (v), U and V are gpr-open sets. Hence X is gpr-T_1.

(iii) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are wg-open sets. Hence X is wg-T_1.

(iv) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. By assumption, there exists a p*gp-open sets U and V such that $x \in U$ but $y \notin U$ and $x \notin V$ and $y \in V$. By Theorem 2.10 (v), V is rwg-open. Hence X is rwg-T_1.

(v) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are πgp-open sets. Hence X is πgp-T_1.

(vi) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are gsp-open sets. Hence X is gsp-T_1.

(vii) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are πgsp-open sets. Hence X is πgsp-T_1.

(viii) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. By assumption, there exists a p*gp-open sets U and V such that $x \in U$ but $y \notin U$ and $x \notin V$ and $y \in V$. By Theorem 2.10 (v), V is pre semi open. Hence X is pre semi-T_1.

(ix) Suppose X is p*gp-T_1 space. Let x and y be two distinct points in X. Since X is p*gp-T_1, there exists a p*gp-open sets containing each but not the other. By Theorem 2.10 (v), U and V are g*p-open sets. Hence X is g*p-T_1.

Theorem 4.3. In any topological space,

(i) Every T_1 space is p*gp-T_1.
(ii) Every regular-T_1 space is p*gp-T_1.
(iii) Every α-T_1 space is p*gp-T_1.
(iv) Every pre-T_1 space is p*gp-T_1.

Proof: (i) Suppose X is T_1 space. Let x and y be two distinct points in X. Since X is T_1, there exists an open sets containing each but not the other. By Theorem 2.10 (i), U and V are p*gp-open sets. Hence X is p*gp-T_1.

(ii) Suppose X is regular-T_1 space. Let x and y be two distinct points in X. Since X is regular-T_1, there exists a regular open sets containing each but not the other. By Theorem 2.10 (ii), U and V are p*gp-open sets. Hence X is p*gp-T_1.

(iii) Suppose X is α-T_1 space. Let x and y be two distinct points in X. Since X is α-T_1, there exists a α-open sets containing each but not the other. By Theorem 2.10 (iii), U and V are p*gp-open sets. Hence X is p*gp-T_1.

(iv) Suppose X is pre-T_1 space. Let x and y be two distinct points in X. Since X is pre-T_1, there exists a pre open sets containing each but not the other. By Theorem 2.10 (iv), U and V are p*gp-open sets. Hence X is p*gp-T_1.

Remark 4.4. The converse of each of the statements in Theorem 4.2 and Theorem 4.3 are not true as shown in the following examples.

Example 4.5. Let X = {a, b, c} and τ ={ϕ, {c}, {b, c}, X}. Then, X is p*gp-T_1 but not T_1 and also gp-T_1, pre semi-T_1 and g*p-T_1 but not p*gp-T_1.

Example 4.6. Let X = {a, b, c} and τ = {ϕ, {c}, {a, b}, X}. Then, X is p*gp-T_1 but not regular-T_1.

Example 4.7. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then, X is p*gp-T_1 but not α-T_1 and also gpr-T_1 and πgsp-T_1 but not p*gp-T_1.

Example 4.8. Let X = {a, b, c} and τ = {ϕ, {c}, X}. Then, X is wg-T_1 and gsp-T_1 but not p*gp-T_1.

Example 4.9. Let X = {a, b, c} and τ ={ϕ, {a}, {b}, {a, b}, X}. Then, X is rwg-T_1 but not p*gp-T_1..

Example 4.10. Let X = {a, b, c} and τ ={ϕ, {a}, {a, b}, X}. Then, X is πgp-T_1 but not p*gp-T_1.

Example 4.11. Let X ={a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then, X is p*gp-T_1 but not pre-T_1.

Theorem 4.12. A topological space (X, τ) is p*gp-T_1 if and only if the singletons are p*gp-closed.

Proof: Let (X, τ) be a p*gp-T_1 space and x be any point of X. Let y ∈ X\{x}. Then x ≠ y and so there exists a p*gp-open set U containing y but not x. That is y ∈ U ⊆ X\{x}. That implies, X\{x} = ∪{U : y ∈ X\{x}}. Since union of p*gp-open set is p*gp-open, X\{x} is p*gp-open containing y but not x. Hence {x} is p*gp-closed in X.

Conversely, suppose {x} is p*gp-closed for every x ∈ X. If x, y ∈ X, x ≠ y, then y ∈ X\{x} and x ∈ X\{y}. Since {x} and {y} are p*gp-closed sets in X, X\{x} and X\{y} are p*gp-open sets in X. Thus, a p*gp-open set containing x but not y and a p*gp-open set containing y but not x. Hence X is a p*gp-T_1 space.

Theorem 4.13. Let (X, τ) be a topological space, then the following statements are equivalent:

(i) X is a p*gp-T_1 space.
(ii) The intersection of all p*gp-open sets containing the set A is A.
(iii)The intersection of all p*gp-open sets containing the point x ∈ X is {x}.

Proof: (i)⇒(ii): Suppose X is a p*gp-T_1 space. Then by Theorem 4.12, each singleton set is p*gp-closed in X. Let A ⊆ X. Then for each x ∈ X\A, {x} is p*gp-closed in X and hence X\{x} is p*gp-open. Clearly, A ⊆ X\{x} for each x ∈ X\A. Therefore A ⊆ ∩{X\{x} : x ∈ X\A}. On the other hand, if y ∉ A then y ∈ X\A and y ∉ X\{y}. Therefore y ∉ ∩{X\{x} : x ∈ X\A} and hence ∩{X\{x}: x ∈ X\A} ⊆ A. Therefore, the intersection of all p*gp-open sets containing the set A is A. This proves (ii).

(ii)⇒(iii): Suppose the intersection of all p*gp-open sets containing the set A is A. Take A = {x}. Then A = {x} = ∩{U: U is p*gp-open and x ∈ U}. Therefore, the intersection of all p*gp-open sets containing the point x ∈ X is {x}. This proves (iii).

(iii)⇒(i): Let x, y ∈ X and y ≠ x. Then y ∉ {x} = ∩{ U : U is p*gp-open and x ∈ U}. Hence there exists a p*gp-open set U containing x but not y. Similarly, there exists a p*gp-open set V containing y but not x. Thus X is a p*gp-T_1 space. This proves (i).
This completes the proof.

Theorem 4.14. Let f : (X, τ) → (Y, σ) be a function.

(i) If f is a p*gp-closed surjection and X is T_1, then Y is p*gp-T_1.
(ii) If f is a p*gp-continuous bijection and Y is T_1, then X is p*gp-T_1.

Proof: (i) Suppose f is p*gp-closed and X is T_1. Let y ∈ Y. Since f is onto, there exists x ∈ X, such that f(x) = y. Since X is T_1, {x} is closed in X. Since f is a p*gp-closed map, f({x}) = {y} is p*gp-closed. Since every singleton set in Y is p*gp-closed, by Theorem 4.12, Y is p*gp-T_1.

(ii) Suppose f : X→Y is a p*gp-continuous bijection and Y is T_1. Let x_1, x_2 ∈ X with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-to-one, $y_1 \neq y_2$. Since Y is T_1, there exist open sets U and V such that y_1 ∈ U but y_2 ∉ U and y_2 ∈ V but y_1 ∉ V. Again since f is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since f is p*gp-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are p*gp-open sets in X. This shows that, X is p*gp-T_1.

5. Pre*generalized pre T_2 Spaces

In this section, we define p*gp-T_2 spaces and investigates their basic properties. Also we define p*gp-$T_{1/2}$ space and p*gp-space and interpret their properties.

Definition 5.1. A topological space (X, τ) is said to be

(i) p*gp-T_2 if for each pair of distinct points x ≠ y in X, there exists two disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V.
(ii) p*gp-$T_{1/2}$ if every p*gp-closed set is pre*closed.
(iii) p*gp-space if every p*gp-open set is open.

Theorem 5.2. In any topological space,

(i) Every p*gp-T_2 space is gp-T_2.
(ii) Every p*gp-T_2 space is gpr-T_2.
(iii) Every p*gp-T_2 space is wg-T_2.
(iv) Every p*gp-T_2 space is rwg-T_2.
(v) Every p*gp-T_2 space is πgp-T_2.
(vi) Every p*gp-T_2 space is gsp-T_2.
(vii) Every p*gp-T_2 is space πgsp-T_2.
(viii) Every p*gp-T_2 space is pre semi-T_2.
(ix) Every p*gp-T_2 is space g*p-T_2.

Proof: (i) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint gp-open sets such that x ∈ U and y ∈ V. Hence X is gp-T_2.

(ii) Assume that X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint gpr-open sets such that x ∈ U and y ∈ V. Hence X is gpr-T_2.

(iii) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint wg-open sets such that x ∈ U and y ∈ V. Hence X is wg-T_2.

(iv) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint rwg-open sets such that x ∈ U and y ∈ V. Hence X is rwg-T_2.

(v) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint πgp-open sets such that x ∈ U and y ∈ V. Hence X is πgp-T_2.

(vi) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint gsp-open sets such that x ∈ U and y ∈ V. Hence X is gsp-T_2.

(vii) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint πgsp-open sets such that x ∈ U and y ∈ V. Hence X is πgsp-T_2.

(viii) Suppose X is p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint pre semi open sets such that x ∈ U and y ∈ V. Hence X is pre semi-T_2.

(ix) Suppose X is a p*gp-T_2 space. Let x and y be two distinct points in X. Since X is p*gp-T_2, there exists disjoint p*gp-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (v), U and V are disjoint g*p-open sets such that x ∈ U and y ∈ V. Hence X is g*p-T_2.

Theorem 5.3. In any topological space,

(i) Every T_2 space is p*gp-T_2.
(ii) Every regular-T_2 space is p*gp-T_2.
(iii) Every α-T_2 space is p*gp-T_2.
(iv) Every pre-T_2 space is p*gp-T_2.

Proof: (i) Suppose X is a T_2 space. Let x and y be two distinct points in X. Since X is T_2, there exists disjoint open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (i), U and V are disjoint p*gp-open sets such that x ∈ U and y ∈ V. Hence X is p*gp-T_2.

(ii) Suppose X is a regular-T_2 space. Let x and y be two distinct points in X. Since X is regular-T_2, there exists disjoint regular open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (ii), U and V are disjoint p*gp-open sets such that x ∈ U and y ∈ V. Hence X is p*gp-T_2.

(iii) Suppose X is an α-T_2 space. Let x and y be two distinct points in X. Since X is α-T_2, there exists disjoint α-open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (iii), U and V are disjoint p*gp-open sets such that x ∈ U and y ∈ V. Hence X is p*gp-T_2.

(iv) Suppose X is a pre-T_2 space. Let x and y be two distinct points in X. Since X is pre-T_2, there exists disjoint pre open sets U and V such that x ∈ U and y ∈ V. By Theorem 2.10 (iv), U and V are disjoint p*gp-open sets such that x ∈ U and y ∈ V. Hence X is p*gp-T_2.

Remark 5.4. The converse of each of the statements in Theorem 5.2 and Theorem 5.3 are not true as shown in the following examples.

Example 5.5. Let X = {a, b, c} and τ ={ϕ, {c}, {b, c}, X}. Then, X is p*gp-T_2 but not T_2 and also gp-T_2, pre semi-T_2 and g*p-T_2 but not p*gp-T_2.

Example 5.6. Let X = {a, b, c} and τ = {ϕ, {c}, {a, b}, X}. Then, X is p*gp-T_2 but not regular-T_2.

Example 5.7. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then, X is p*gp-T_2 but not α-T_2 and also gpr-T_2 and πgsp-T_2 but not p*gp-T_2.

Example 5.8. Let X={a, b, c} and τ = {ϕ, {c}, X}. Then, X is wg-T_2 and gsp-T_2 but not p*gp-T_2.

Example 5.9. Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then, X is rwg-T_2 but not p*gp-T_2.

Example 5.10. Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, X}. Then, X is πgp-T_2 but not p*gp-T_2.

Example 5.11. Let X ={a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then, X is p*gp-T_2 but not pre-T_2.

Theorem 5.12. The following statements are equivalent for a topological space X.

(i) X is p*gp-T_2 space.

(ii) Let x ∈ X. For each y ≠ x, there exists a p*gp-open set U containing x such that y ∉ p*gpcl(U).

(iii) For each x ∈ X, ∩{p*gpcl(U) : U ∈ p*gp-O(X) and x ∈ U} = {x}.

Proof: (i)⇒(ii): Suppose X is p*gp-T_2. Let x ∈ X and y ∈ X with x ≠ y. Then there exists disjoints p*gp-open sets U and V containing x and y respectively. That implies, U ⊆ X\V. Since V is p*gp-open, X\V is p*gp-closed containing U. Hence p*gpcl(U) ⊆ X\V. Since y ∈ V, y ∉ X\V and hence y ∉ p*gpcl(U). This proves (ii).

(ii)⇒(iii): Assume that x ∈ X, for each y ≠ x, there exists a p*gp-open set U containing x such that y ∉ p*gpcl(U). Now, prove that ∩{p*gpcl(U) : U ∈ p*gp-O(X) and x ∈ U} = {x}. Suppose not. Then there exists an element y ≠ x in X such that y ∈ p*gpcl(U) for every p*gp-open set U containing x. This is a contradiction to our assumption. This proves (iii).

(iii)⇒(i): Let x ∈ X and y ∈ X with x ≠ y. Then by our assumption, there exists a p*gp-open set U containing x such that y ∉ p*gpcl(U). Let V = X\p*gpcl(U). Then V is p*gp-open set containing y. Also x ∈ U and U∩V = ϕ. Thus, a disjoint p*gp-open sets U and V containing x and y respectively. Hence X is p*gp-T_2 space.

This completes the proof.

Theorem 5.13. Let (X, τ) be a topological space, then the following statements are true.

(i) Every p*gp-T_2 space is p*gp-T_1.

(ii) Every p*gp-space is p*gp-$T_{1/2}$.

Proof: (i) Let (X, τ) be p*gp-T_2. Then for each distinct points x, y in X, there exists two disjoint p*gp-open sets U and V containing x and y respectively. That is, for each distinct points x, y in X, there exists two p*gp-open sets U and V such that x ∈ U but y ∉ U and x ∉ V and y ∈ V. Hence X is p*gp-T_1.

(ii) Let (X, τ) be a p*gp-space and A be any p*gp-closed set in X. Then X\A is p*gp-open in X. Since X is p*gp-space, X\A is open in X. By Theorem 2.6 (ii), X\A is pre*open in X and hence A is pre*closed. Since the p*gp-closed set is arbitrary, the space X is p*gp-$T_{1/2}$.

Theorem 5.14. Let f : (X, τ) → (Y, σ) be a bijection.
(i) If f is a p*gp-open and X is T_2, then Y is p*gp-T_2.
(ii) If f is a p*gp-continuous and Y is T_2, then X is p*gp-T_2.

Proof: Let f : (X, τ) → (Y, σ) be a bijection.

(i) Suppose f is p*gp-open and X is T_2. Let $y_1 \neq y_2 \in$ Y. Since f is a bijection, there exists x_1, x_2 in X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ with $x_1 \neq x_2$. Since X is T_2, there exists disjoint open sets U and V in X such that $x_1 \in$ U and $x_2 \in$ V. Since f is p*gp-open, f(U) and f(V) are p*gp-open in Y such that $y_1 = f(x_1) \in f$(U) and $y_2 = f(x_2) \in f$(V). Again since f is a bijection, f(U) and f(V) are disjoint in Y. Thus Y is p*gp-T_2.

(ii) Suppose f : (X, τ) → (Y, σ) is a p*gp-continuous and Y is T_2. Let x_1, $x_2 \in$ X with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-to-one, $y_1 \neq y_2$. Since Y is T_2, there exists disjoint open sets U and V containing y_1 and y_2 respectively. Since f is p*gp-continuous, f^{-1}(U) and f^{-1}(V) are disjoint p*gp-open sets in X containing x_1 and x_2 respectively. Thus X is p*gp-T_2.

Theorem 5.15. If a topological space (X, τ) is p*gp-$T_{1/2}$ then each singleton {x} of X is either pre*closed or pre*open.

Proof: Let (X, τ) be a p*gp-$T_{1/2}$ space.

Case (i): Suppose {x} is not pre*closed. Then X\{x} is not pre*open. By Theorem 2.14, X\{x} is p*gp-closed. Since X is a p*gp-$T_{1/2}$ space, X\{x} is pre*closed and hence {x} is pre*open.

Case (ii): Suppose {x} is not pre*open. Then X\{x} is not pre*closed. By Theorem 2.15, X\{x} is p*gp-open. Since X is a p*gp-$T_{1/2}$ space, X\{x} is pre*open and hence {x} is pre*closed.

6. Pre*generalized pre D_k Spaces

In this section, we introduce p*gp-D_k spaces and investigates their basic properties.

Definition 6.1. A subset A of a topological space X is called a p*gp-difference (briefly p*gp-D) set if there are U, V ∈ p*gp-O(X) such that U ≠ X and A = U\V.

Theorem 6.2. Every proper p*gp-open set is a p*gp-D set.

Proof: Let A be any proper p*gp-open subset of a topological space X. Take U = A and V = ϕ. Then A = U\V and U ≠ X. Hence A is p*gp-D set.

Remark 6.3. The converse of the above theorem is not true. It is shown in the following example.

Example 6.4. Let X = {a, b, c} be given the topology τ = {ϕ, {a, b}, X}. Then p*gp-O(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Therefore the subset {c} is a p*gp-D set but not a proper p*gp-open set.

Definition 6.5. A topological space (X, τ) is said to be
(i) p*gp-D_0 if for any pair of distinct points x and y of X there exists a p*gp-D set of X containing x but not y or a p*gp-D set of X containing y but not x.
(ii) p*gp-D_1 if for any pair of distinct points x and y of X there exists a p*gp-D sets U and V such that x ∈ U but y ∉ U and y ∈ V but x ∉ V.

(iii) p*gp-D_2 if for any pair of distinct points x and y of X there exists a p*gp-D sets G and E of X containing x and y respectively.

The following theorem supports the above definition.

Theorem 6.6. Let (X, τ) be a topological space, then the following statements are true.

(i) If (X, τ) is p*gp-T_k, then it is p*gp-D_k, for k = 0, 1, 2.

(ii) If (X, τ) is p*gp-D_k, then it is p*gp-D_{k-1}, for k = 1, 2.

Proof: (i) First, prove that the result for k = 0. Suppose (X, τ) is p*gp-T_0. Then for each pair of distinct points x, y in X, there exists a p*gp-open set U such that either $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$. By Theorem 6.2, U is p*gp-D-set in X. Then for each pair of distinct points x, y in X, there exists a p*gp-open set U such that either $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$. Hence (X, τ) is a p*gp-D_0 space.

Similarly, it is prove that every p*gp-T_k space is p*gp-D_k, for k = 1, 2.

(ii) Let k = 2. Suppose (X, τ) is a p*gp-D_2 space. Then for any pair of distinct points x and y of X, there exists disjoint p*gp-D sets U and V of X containing x and y respectively. That is for any pair of distinct points x and y of X, there exists a p*gp-D sets U of X containing x but not y and a p*gp-D sets V of X containing y but not x. Hence (X, τ) is a p*gp-D_1 space.

Similarly, it is prove that every p*g-D_1 space is a p*gp-D_0 space.

Further, p*gp-D_0 is equivalent to p*gp-T_0 as shown in the following theorem.

Theorem 6.7. A topological space X is p*gp-D_0 if and only if it is p*gp-T_0.

Proof: Suppose that X is p*gp-D_0. Then for any pair of distinct points x and y of X, there is a p*gp-D sets G containing x or y, say x but not y. Since G is p*gp-D set, then there are two p*gp-open sets U_1 and U_2 such that $U_1 \neq X$ and $G = U_1 \setminus U_2$. Since $x \in G$ and $y \notin G$, then $x \in U_1$. For $y \notin G$, we have two cases,

(i) $y \notin U_1$.

(ii) $y \in U_1$ and $y \in U_2$.

In case (i), $x \in U_1$ and $y \notin U_1$. In case (ii), $y \in U_2$ and $x \notin U_2$. Thus in both cases a distinct points x and y in X, there exists a p*gp-open set U_1 containing x but not y or a p*gp-open set U_2 containing y but not x. Hence X is p*gp-T_0.

Conversely, suppose X is p*gp-T_0. Then by Theorem 6.6 (i), X is p*gp-D_0.

The next theorem indicates that p*gp-D_1 is equivalent to p*gp-D_2.

Theorem 6.8. A topological space X is p*gp-D_1 if and only if it is p*gp-D_2.

Proof: Suppose X is p*gp-D_1. Let $x, y \in X$ with $x \neq y$. Then there exists p*gp-D sets G_1, G_2 in X such that $x \in G_1$, $y \notin G_1$ and $y \in G_2$, $x \notin G_2$. Since G_1, G_2 are p*gp-D sets, then $G_1 = U_1 \setminus U_2$ and $G_2 = U_3 \setminus U_4$, where U_1, U_2, U_3 and U_4 are p*gp-open sets in X. From $x \notin G_2$, it follows that either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$.

Discuss the two cases separately.

(i) Suppose $x \notin U_3$. By $y \notin G_1$ we have two sub-cases: (a) $y \notin U_1$. Since $x \in U_1 \setminus U_2$, it follows that $x \in U_1 \setminus (U_2 \cup U_3)$, and since $y \in U_3 \setminus U_4$, $y \in U_3 \setminus (U_1 \cup U_4)$. Since by Theorem 2.10 (vi),

$U_2 \cup U_3$ and $U_1 \cup U_4$ are p*gp-open sets. Also $U_1 \backslash (U_2 \cup U_3) \cap U_3 \backslash (U_1 \cup U_4) = \phi$. Thus, a disjoint p*gp-D sets $U_1 \backslash (U_2 \cup U_3)$ and $U_3 \backslash (U_1 \cup U_4)$ containing x and y respectively.

(b) If $y \in U_1$ and $y \in U_2$, then $x \in U_1 \backslash U_2$ and $y \in U_2$. Also $(U_1 \backslash U_2) \cap U_2 = \phi$. Thus, a disjoint p*gp-D sets $U_1 \backslash U_2$ and U_2 containing x and y respectively.

(ii) Suppose $x \in U_3$ and $x \in U_4$, $y \in U_3 \backslash U_4$ and $x \in U_4$. Hence $(U_3 \backslash U_4) \cap U_4 = \phi$. Thus, a disjoint p*gp-D sets $U_3 \backslash U_4$ and U_4 containing x and y respectively. Hence X is p*gp-D_2.

Conversely, suppose X is p*gp-D_2. Then by Theorem 6.6 (ii), X is p*gp-D_1.

Definition 6.9. A point $x \in X$ which has only X as the p*gp-neighbourhood is called a p*gp-neat point.

Theorem 6.10. For a p*gp-T_0 topological space (X, τ) the following are equivalent:

(i) X is p*gp-D_1.

(ii) X has no p*gp-neat point.

Proof: (i) ⇒ (ii): Suppose X is p*gp-D_1, then each point x of X is contained in a p*gp-D set A = U\V and thus in U. By definition $U \neq X$. This implies that x is not a p*gp-neat point. That is X has no p*gp-neat point.

(ii) ⇒ (i): Suppose X has no p*gp-neat point. Let x and y be distinct points in X. Since X is p*gp-T_0, there exists a p*gp-open set U containing x or y, say x. Since $y \notin U$, $U \neq X$, by Theorem 6.2, U is a p*gp-D set. Since X has no p*gp-neat point, y is not a p*gp-neat point. This means that there exists a p*gp-neighbourhood V of y such that $V \neq X$. Since V is a p*gp-neighbourhood of y, there exists a p*gp-open set G such that $y \in G \subseteq V$. Thus $y \in G \backslash U$ but not x. Also G\U is a p*gp-D set. Hence X is a p*gp-D_1 space.

Corollary 6.11. A p*gp-T_0 space X is not p*gp-D_1 if and only if there is a unique p*gp-neat point in X.

Proof: Let X be a p*gp-T_0 space. By Theorem 6.10, X is not p*gp-D_1, then X has p*gp-neat points. Only prove that, the uniqueness of the p*gp-neat point. If x and y are two p*gp-neat points in X. Since X is p*gp-T_0, at least one of x and y, say x, has a p*gp-open set containing x but not y, then U is a p*gp-neighbourhood of x and $U \neq X$. Therefore x is not a p*gp-neat point which is a contradiction to x is a p*gp-neat point. Hence x = y.

7. Conclusion

The spaces namely p*gp-T_k space, p*gp-D_k space and p*gp-neat points are studied using p*gp-closed sets. The relationship of p*gp-T_k spaces with p*gp-D_K spaces is established. Finally, some of their fundamental properties are discussed.

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