Fuzzy Tri-Magic Labeling of Isomorphic Caterpillar of Diameter 5 – Paper 1

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Abstract: *Let G be a finite, simple, undirected and non-trivial graph. A fuzzy graph is said to admit tri-magic labeling if the number of magic membership values K_i's, $1 \leq i \leq 3$ differ by atmost 1 and $\left|K_i - K_j\right| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$, $r \geq 2$. The fuzzy graph which admits a tri-magic labeling is called a fuzzy tri-magic labeling graph. The fuzzy tri-magic labeling graphs are denoted by $\tilde{T}m_0G$. Caterpillar is a tree in which all the vertices are within distance one of a central path In this paper it is proved that isomorphic caterpillar of diameter 5 are Fuzzy tri-magic.*

Keywords: Fuzzy Tri-Magic Labeling, Diameter of a graph, Caterpillar graph.

1. INTRODUCTION

The graphs considered here are finite, simple, undirected and non trivial [1]. Graph theory has a good development in the graph labeling and has a broad range of applications [2]. Fuzzy is a newly emerging mathematical framework to exhibit the phenomenon of uncertainty in real life tribulations. A fuzzy set is defined mathematically by assigning a value to each possible individual in the universe of discourse, representing its grade or membership which corresponds to the degree to which that individual is similar or compatible with the concept represented the fuzzy set. In this paper it is proved that isomorphic caterpillar of diameter 5 are Fuzzy tri-magic.

Definition 1.1 Fuzzy graph

A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 1.2 Fuzzy Labeling

Let $G = (V,E)$ be a graph, the fuzzy graph $G:(\sigma,\mu)$ is said to have a fuzzy labeling, if $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is bijective such that the membership value of edges and vertices is distinct and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

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Definition 1.3 Magic membership value (MMV) [3]

Let $G:(\sigma,\mu)$ be a fuzzy graph; the induced map $g:E(G)\rightarrow[0,1]$ defined by $g(uv)=\sigma(u)+\mu(uv)+\sigma(v)$ is said to be a magic membership value. It is denoted by MMV.

Definition 1.4 Fuzzy tri-magic labeling

A fuzzy graph is said to admit tri-magic labeling if the magic membership values K_i's, $1\leq i\leq 3$ are constants where number of K_i's and K_j's differ by at most 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1\leq i,j\leq 3$, $r\geq 2$.

Definition 1.5 Fuzzy tri-magic labeling graph

A fuzzy labeling graph which admits a tri-magic labeling is called a fuzzy tri-magic labeling graph. The fuzzy tri-magic labeling graphs are denoted by $\tilde{T}m_0G$.

Definition 1.6 Diameter of a graph

The maximum distance between two vertices of a tree is called the diameter of a graph.

Definition 1.7 Caterpillar graph [4, 5]

A caterpillar graph G, is a tree having a chordless path P_t on t vertices, called central path, which contains at least one end point of every edge. Vertices connecting the leaves with the central path are called support vertices.

Notation 1.8

Let $\mathcal{T}^{s_0}$ is a path P_s with no attachments.

Notation 1.9

Let $\mathcal{T}_{t_1}^{s_1}$ is a tree obtained by attaching n pendant edges to the path P_s at the vertex v_{t_1+1} internal vertices of P_s.

Notation 1.10

Let $\mathcal{T}_{t_1,t_2}^{s_1}$ is a tree obtained by attaching m pendant edges and n pendant edges to the path P_s at the vertices v_{t_1+1} and v_{t_2+1} respectively.

Notation 1.11

Let $\mathcal{T}_{t_1,t_2,t_3}^{s_1}$ is a tree obtained by attaching m pendant edges, n pendant edges and a pendant edges to the path P_s at the vertices v_{t_1+1}, v_{t_2+1} and v_{t_3+1} respectively.

Notation 1.12

Let $\mathcal{T}_{t_1,t_2,t_3,t_4}^{s_1}$ is a tree obtained by attaching m pendant edges, n pendant edges, a pendant edges and b pendant edges to the path P_s at the vertices v_{t_1+1}, v_{t_2+1}, v_{t_3+1} and v_{t_4+1} respectively.

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Remark:

In $\mathcal{T}^6$, isomorphic graphs and non-isomorphic graphs are identified and it is given below:

(i) Isomorphic graphs:

$\mathcal{T}_2^6$ & $\mathcal{T}_5^6$, $\mathcal{T}_3^6$ & $\mathcal{T}_4^6$, $\mathcal{T}_{2,3}^6$ & $\mathcal{T}_{4,5}^6$, $\mathcal{T}_{2,4}^6$ & $\mathcal{T}_{3,5}^6$, $\mathcal{T}_{2,3,4}^6$ & $\mathcal{T}_{3,4,5}^6$, $\mathcal{T}_{2,3,5}^6$ & $\mathcal{T}_{2,4,5}^6$

(i) Non-isomorphic graphs:

$\mathcal{T}_{3,4}^6$, $\mathcal{T}_{2,5}^6$, $\mathcal{T}_{2,3,4,5}^6$

2. MAIN RESULT

Theorem 2.1: The caterpillar graph $\mathcal{T}_2{}^6$ of diameter 5 admits fuzzy tri-magic labeling.

Proof:

Let G be a caterpillar graph $\mathcal{T}_2{}^6$ of diameter 5. $|V(G)| =$ n + 6 and $|E(G)| =$ n + 5.

Let the vertex set and edge set of $\mathcal{T}_2{}^6$ be

$V(G) = \{ v_j : 1 \leq j \leq 6\} \cup \{x_j : 1 \leq j \leq n\}$ and

$E(G) = \{v_j v_{j+1} : 1 \leq j \leq 5\} \cup \{v_2 x_j : 1 \leq j \leq n\}$

Let $r \geq 2$ be any positive integer.

Define $\sigma : V \rightarrow [0,1]$ such that

$\sigma(v_j) = \frac{2n+17-j}{10^r}$ for $1 \leq j \leq 6$

Define $\mu : V \times V \rightarrow [0,1]$ by

$\mu(v_1 v_2) = \frac{1}{10^r}$

$\mu(v_2 v_3) = \frac{3}{10^r}$

$\mu(v_3 v_4) = \frac{4}{10^r}$

$\mu(v_4 v_5) = \frac{6}{10^r}$

$\mu(v_5 v_6) = \frac{7}{10^r}$

$\mu(v_2 x_j) = (7 + j) \frac{1}{10^r}$ for $1 \leq j \leq n$

Case (i) If $n \equiv 0(mod\ 3)$

Define $\sigma : V \rightarrow [0,1]$ such that

$\sigma(x_j) = (2n + 10 - j) \frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\sigma(x_j) = (2n + 9 - j) \frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\sigma(x_j) = (2n + 8 - j) \frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n$

By the definition of MMV:

$K_1 = \frac{4n+32}{10^r}$, $K_2 = \frac{4n+31}{10^r}$ and $K_3 = \frac{4n+30}{10^r}$

$|K_1| = |K_2| = \frac{n+6}{3}$ and $|K_3| = \frac{n+3}{3}$

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Case (ii) If $n \equiv 1(mod\ 3)$

$\sigma(x_j) = (2n + 10 - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n-1}{3}$

$\sigma(x_j) = (2n + 9 - j)\ \frac{1}{10^r}$ for $\frac{n-1}{3} + 1 \leq j \leq \frac{2(n-1)}{3}$

$\sigma(x_j) = (2n + 8 - j)\ \frac{1}{10^r}$ for $\frac{2(n-1)}{3} + 1 \leq j \leq n$

By the definition of MMV:

$K_1 = \frac{4n+32}{10^r}, K_2 = \frac{4n+31}{10^r}\ and\ K_3 = \frac{4n+30}{10^r}$

$|K_1| = |K_2| = |K_3| = \frac{n+5}{3}$

Case (iii) If $n \equiv 2(mod\ 3)$

$\sigma(x_j) = (2n + 10 - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n-2}{3}$

$\sigma(x_j) = (2n + 9 - j)\ \frac{1}{10^r}$ for $\frac{n-2}{3} + 1 \leq j \leq \frac{2n-1}{3}$

$\sigma(x_j) = (2n + 8 - j)\ \frac{1}{10^r}$ for $\frac{2n-1}{3} + 1 \leq j \leq n$

By the definition of MMV:

$K_1 = \frac{4n+32}{10^r}, K_2 = \frac{4n+31}{10^r}\ and\ K_3 = \frac{4n+30}{10^r}$

$|K_1| = |K_3| = \frac{n+4}{3}$ and $|K_2| = \frac{n+7}{3}$

The magic membership values of K_i's, $1 \leq i \leq 3$ are tabulated below

Nature of n	**Edges**	**MMV $K_i's$, 1≤ i ≤ 3**	**Number of $K_i's$, 1≤ i ≤ 3**
$n \equiv 0(mod\ 3)$	$g(v_2x_j)$ if $1 \leq j \leq \frac{n}{3}$ $g(v_1v_2),\ g(v_2v_3)$	$(4n + 32)\ \frac{1}{10^r}$ for i = 1	$\frac{n+6}{3}$ for i = 1
	$g(v_2x_j)$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$ $g(v_2v_4),\ g(v_4v_5)$	$(4n + 31)\ \frac{1}{10^r}$ for i = 2	$\frac{n+6}{3}$ for i = 2
	$g(v_2x_j)$ if $\frac{2n}{3} + 1 \leq j \leq n$ $g(v_5v_6)$	$(4n + 30)\ \frac{1}{10^r}$ for i = 3	$\frac{n+3}{3}$ for i = 3
	$g(v_2x_j)$ if $1 \leq j \leq \frac{n-1}{3}$ $g(v_1v_2),\ g(v_2v_3)$	$(4n + 32)\ \frac{1}{10^r}$ for i = 1	$\frac{n+5}{3}$ for i = 1

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$n \equiv 1(mod\ 3)$	$g(v_2x_j)$ if $\frac{n-1}{3}+1 \le j \le \frac{2(n-1)}{3}$ $g(v_3v_4),\ g(v_4v_5)$	$(4n+31)\ \frac{1}{10^r}$ for i = 2	$\frac{n+5}{3}$ for i = 2
	$g(v_2x_j)$ if $\frac{2(n-1)}{3}+1 \le j \le n$ $g(v_5v_6)$	$(4n+30)\ \frac{1}{10^r}$ for i = 3	$\frac{n+5}{3}$ for i = 3
$n \equiv 2(mod\ 3)$	$g(v_2x_j)$ if $1 \le j \le \frac{n-2}{3}$ $g(v_1v_2),\ g(v_2v_3)$	$(4n+32)\ \frac{1}{10^r}$ for i = 1	$\frac{n+4}{3}$ for i = 1
	$g(v_2x_j)$ if $\frac{n-2}{3}+1 \le j \le \frac{2n-1}{3}$ $g(v_3v_4),\ g(v_4v_5)$	$(4n+31)\ \frac{1}{10^r}$ for i = 2	$\frac{n+7}{3}$ for i = 2
	$g(v_2x_j)$ if $\frac{2n-1}{3}+1 \le j \le n$ $g(v_5v_6)$	$(4n+30)\ \frac{1}{10^r}$ for i = 3	$\frac{n+4}{3}$ for i = 3

Table 2.1.1

Hence the maximum difference between the number of K_i's is 1 and

$|K_i - K_j| \le \frac{2}{10^r}$ for $1 \le i, j \le 3$. Hence the caterpillar graph $\mathcal{T}_2^{\,6}$ admits fuzzy tri-magic labeling.

Theorem 2.2: The path $\mathcal{T}_0^{\,6}$ of diameter 5 admits fuzzy tri-magic labeling.

Proof:

The proof of this theorem is similar to the theorem 2.1 by taking n = 0.

Where the vertex set and edge set of $\mathcal{T}_0^{\,6}$ be $V(G) = \{v_j : 1 \le j \le 6\}$ and

$E(G) = \{v_jv_{j+1} : 1 \le j \le 5\}$

Theorem 2.3: The caterpillar graph $\mathcal{T}_3^{\,6}$ of diameter 5 admits fuzzy tri-magic labeling.

Proof:

Let G be a caterpillar graph $\mathcal{T}_3^{\,6}$ of diameter 5. $|V(G)| = n + 6$ and $|E(G)| = n + 5$.

Let the vertex set and edge set of $\mathcal{T}_3^{\,6}$ be

$V(G) = \{v_j : 1 \le j \le 6\} \cup \{x_j : 1 \le j \le n\}$ and

$E(G) = \{v_jv_{j+1} : 1 \le j \le 5\} \cup \{v_3x_j : 1 \le j \le n\}$

Let $r \ge 2$ be any positive integer.

Define $\sigma : V \to [0,1]$ such that

$\sigma(v_j) = \frac{2n+16-j}{10^r}$ for $1 \le j \le 6$

Define $\mu : V \times V \to [0,1]$ by

$\mu(v_1v_2) = \frac{1}{10^r}$

$\mu(v_2v_3) = \frac{3}{10^r}$

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$\mu(v_3v_4) = \frac{4}{10^r}$

$\mu(v_4v_5) = \frac{6}{10^r}$

$\mu(v_5v_6) = \frac{7}{10^r}$

$\mu(v_3x_j) = (7+j)\,\frac{1}{10^r}$ for $1 \le j \le n$

Case (i) If $n \equiv 0(mod\ 3)$

Define $\sigma : V \to [0,1]$ such that

$\sigma(x_j) = (2n+10-j)\,\frac{1}{10^r}$ for $1 \le j \le \frac{n}{3}$

$\sigma(x_j) = (2n+9-j)\,\frac{1}{10^r}$ for $\frac{n}{3}+1 \le j \le \frac{2n}{3}$

$\sigma(x_j) = (2n+8-j)\,\frac{1}{10^r}$ for $\frac{2n}{3}+1 \le j \le n$

By the definition of MMV:

$K_1 = \frac{4n+30}{10^r},\ K_2 = \frac{4n+29}{10^r}\ and\ K_3 = \frac{4n+28}{10^r}$

$|K_1| = |K_2| = \frac{n+6}{3}\ and\ |K_3| = \frac{n+3}{3}$

Case (ii) If $n \equiv 1(mod\ 3)$

$\sigma(x_j) = (2n+10-j)\,\frac{1}{10^r}$ for $1 \le j \le \frac{n-1}{3}$

$\sigma(x_j) = (2n+9-j)\,\frac{1}{10^r}$ for $\frac{n-1}{3}+1 \le j \le \frac{2(n-1)}{3}$

$\sigma(x_j) = (2n+8-j)\,\frac{1}{10^r}$ for $\frac{2(n-1)}{3}+1 \le j \le n$

By the definition of MMV:

$K_1 = \frac{4n+30}{10^r},\ K_2 = \frac{4n+29}{10^r}\ and\ K_3 = \frac{4n+28}{10^r}$

$|K_1| = |K_2| = |K_3| = \frac{n+5}{3}$

Case (iii) If $n \equiv 2(mod\ 3)$

$\sigma(x_j) = (2n+10-j)\,\frac{1}{10^r}$ for $1 \le j \le \frac{n-2}{3}$

$\sigma(x_j) = (2n+9-j)\,\frac{1}{10^r}$ for $\frac{n-2}{3}+1 \le j \le \frac{2n-1}{3}$

$\sigma(x_j) = (2n+8-j)\,\frac{1}{10^r}$ for $\frac{2n-1}{3}+1 \le j \le n$

By the definition of MMV:

$K_1 = \frac{4n+30}{10^r},\ K_2 = \frac{4n+29}{10^r}\ and\ K_3 = \frac{4n+28}{10^r}$

$|K_1| = |K_3| = \frac{n+4}{3}$ and $|K_2| = \frac{n+7}{3}$

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The magic membership values of K_i's, $1 \leq i \leq 3$ are tabulated below

Nature of n	Edges	MMV $K_i's$, $1 \leq i \leq 3$	Number of $K_i's$, $1 \leq i \leq 3$
$n \equiv 0(mod\ 3)$	$g(v_3x_j)$ if $1 \leq j \leq \frac{n}{3}$ $g(v_1v_2)$, $g(v_2v_3)$	$(4n+30)\ \frac{1}{10^r}$ for i = 1	$\frac{n+6}{3}$ for i = 1
	$g(v_3x_j)$ if $\frac{n}{3}+1 \leq j \leq \frac{2n}{3}$ $g(v_3v_4)$, $g(v_4v_5)$	$(4n+29)\ \frac{1}{10^r}$ for i = 2	$\frac{n+6}{3}$ for i = 2
	$g(v_3x_j)$ if $\frac{2n}{3}+1 \leq j \leq n$ $g(v_5v_6)$	$(4n+28)\ \frac{1}{10^r}$ for i = 3	$\frac{n+3}{3}$ for i = 3
$n \equiv 1(mod\ 3)$	$g(v_3x_j)$ if $1 \leq j \leq \frac{n-1}{3}$ $g(v_1v_2)$, $g(v_2v_3)$	$(4n+30)\ \frac{1}{10^r}$ for i = 1	$\frac{n+5}{3}$ for i = 1
	$g(v_3x_j)$ if $\frac{n-1}{3}+1 \leq j \leq \frac{2(n-1)}{3}$ $g(v_3v_4)$, $g(v_4v_5)$	$(4n+29)\ \frac{1}{10^r}$ for i = 2	$\frac{n+5}{3}$ for i = 2
	$g(v_3x_j)$ if $\frac{2(n-1)}{3}+1 \leq j \leq n$ $g(v_5v_6)$	$(4n+28)\ \frac{1}{10^r}$ for i = 3	$\frac{n+5}{3}$ for i = 3
$n \equiv 2(mod\ 3)$	$g(v_3x_j)$ if $1 \leq j \leq \frac{n-2}{3}$ $g(v_1v_2)$, $g(v_2v_3)$	$(4n+30)\ \frac{1}{10^r}$ for i = 1	$\frac{n+4}{3}$ for i = 1
	$g(v_3x_j)$ if $\frac{n-2}{3}+1 \leq j \leq \frac{2n-1}{3}$ $g(v_3v_4)$, $g(v_4v_5)$	$(4n+29)\ \frac{1}{10^r}$ for i = 2	$\frac{n+7}{3}$ for i = 2
	$g(v_3x_j)$ if $\frac{2n-1}{3}+1 \leq j \leq n$ $g(v_5v_6)$	$(4n+28)\ \frac{1}{10^r}$ for i = 3	$\frac{n+4}{3}$ for i = 3

Table 2.3.1

Hence the maximum difference between the number of K_i's is 1 and

$|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$. Hence the caterpillar graph $\mathcal{J}_3^6$ admits fuzzy tri-magic labeling.

CONCLUSION

In this paper, we have shown that the isomorphic caterpillars of diameter 5 are fuzzy tri-magic. All the remaining cases of caterpillar of diameter 5 are also done by us.

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